SEISMIC RESPONSE OF STRUCTURES WITH UNILATERAL CONTACT CONDITIONS AT THE FOUNDATION

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INTRODUCTION

In the paper the dynamic response of framed structures with surface foundation supported on soil which is capable of supporting only compressive stresses (unilateral frictionless contact conditions) is studied. The structures are supposed to be elastic with infinitesimal displacements and strains.

A method is presented for the numerical solution of this problem. Two or three dimensional structures, discrete or discretized by the finite element method are solved using also a temporal discretization. For each time step an elimination of the internal degrees of freedom gives rise to a Linear Complementarity Problem (L.C.P.) on the boundary for only a small number of unknowns, which are the unknown unilateral displacements of the time step under consideration.

A parametric numerical analysis of 2D and 3D structures under seismic excitations is made with unilateral and bilateral contact conditions. Some characteristic results of this analysis are presented in the paper.

FORMULATION OF THE PROBLEM

We consider first the system of the bodies Ω_0 for which only the kinematical constraints on Γ_u hold. The dynamic equations of equilibrium of this system at any time are written as:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{p}(t) \tag{1}$$

In (1) $\boldsymbol{K}$ is the $6n \times 6n$ stiffness matrix, $\boldsymbol{M}$ is the $6n \times 6n$ positive definite mass matrix, $\boldsymbol{C}$ is the $6n \times 6n$ damping matrix, $\boldsymbol{p}(t)$ is the $6n$ vector of the nodal forces, and $\boldsymbol{u}$, $\dot{\boldsymbol{u}}$, $\ddot{\boldsymbol{u}}$ the $6n$ vectors of the nodal displacements, velocities and accelerations.

As long as contact-impact does not take place at any boundary point, the motion is fully described by the linear differential equations (1) and by the "initial" conditions at time $t=t_A$: $u=u_A$ and $\dot{u}=\dot{u}_A$. The "initial" conditions are introduced at time $t=0$ and at any time step t_A for which discontinuity of the velocities occurs due to contact-impact.

For the solution of the problem also an appropriate time discretization algorithm is used. Here the simple one-step algorithm of implicit type proposed by Zienkiewicz, Wood and Taylor (1980) is used. On the basis of this algorithm the differential equations of motion (1) of the system Ω_o are converted to a set of algebraic equations which for the discrete time interval $(t-\Delta t)\div t$ takes the form:

$$\overline{\boldsymbol{K}} \cdot \boldsymbol{u}_t = \overline{\boldsymbol{P}}_t \tag{2}$$

After the above discretization, the unilateral contact conditions at the boundaries Γ_S are introduced. At the m pairs of the discrete contact points, fictitious semi-rigid unilateral bonds with infinitesimal size are introduced, which can carry only compressive stresses. Each bond i has a direction normal to the contact surface and connects the adjacent nodes k and l. Denoting by:

s_i the stress of the bond i (normal contact reaction at the pair i),

e_i the imposed strain of the bond i corresponding to s_i (relative displacements of the node pair k and l), and

h_i the initial gap between the node pair i,

the following relations hold:

$$-s_i \geq 0, \qquad \varepsilon_i = e_i + h_i = \boldsymbol{G}_i^T\ \boldsymbol{u} + h_i \geq 0, \qquad -s_i \cdot \varepsilon_i = 0. \tag{3}$$

where

$\boldsymbol{G}_i^T = [\boldsymbol{o} \ldots \boldsymbol{g}_{ik} \ldots \boldsymbol{g}_{il} \ldots \boldsymbol{o}]$

is the $1 \times n$ strain-displacement matrix of the bond i connecting nodes k and l,

$-\boldsymbol{g}_{ik} = \boldsymbol{g}_{il} = \boldsymbol{g}_i = [n_{ix}, n_{iy}, n_{iz}, 0, 0, 0]$

and $n_i=[n_{ix}, n_{iy}, n_{iz}]$ is the unit vector of the direction cosines which is normal to the bodies A and B at the location of the bond i. For the total number of the m unilateral bonds, at the time step t, the following matrix relations can be written:

$$\boldsymbol{\varepsilon} = \boldsymbol{G}^T\boldsymbol{u} + \boldsymbol{h} \geq \boldsymbol{0}, \qquad -\boldsymbol{s} \geq \boldsymbol{0}, \qquad \boldsymbol{s}^T \cdot \boldsymbol{\varepsilon} = 0 \tag{4}$$

using the vectors: $\mathbf{s}_t = [\ \mathrm{s}_{1t}\ ..\ \mathrm{s}_{mt}\ ]$, $\mathrm{e}_t = [\ \mathrm{e}_{1t}\ ..\ \mathrm{e}_{mt}\ ]$, $\mathbf{h}_t = [\ \mathrm{h}_{1t}\ ..\ \mathrm{h}_{mt}\ ]$, and $\boldsymbol{G}^T = [\ \boldsymbol{G}_1^T \quad \boldsymbol{G}_2^T\ .\ .\ .\ \boldsymbol{G}_m^T\ ]$.

The unilateral contact kinematic conditions $\boldsymbol{\varepsilon}_t=\boldsymbol{G}^T\boldsymbol{u}_t+\boldsymbol{h}$ will be taken into account together with the equations of dynamic equilibrium (2), throught the technique of Langrange multipliers. Accordingly to this technique if (2) are the dynamic equilibrium equations without any constrains, $\boldsymbol{G}^T\boldsymbol{u}_t-\boldsymbol{\varepsilon}_t+\boldsymbol{h}=\boldsymbol{0}$ are the kinematic constraints, and $\boldsymbol{s}_t$ are the reactions corresponding to the kinematic constraints, then (see e.g. Washizu 1975):

$$\overline{\boldsymbol{K}}\boldsymbol{u}_t + \boldsymbol{G}\boldsymbol{s}_t = \overline{\boldsymbol{p}}_t, \qquad \boldsymbol{G}^T\boldsymbol{u}_t = \boldsymbol{\varepsilon}_t - \boldsymbol{h} \tag{5}$$

Since the matrix $\boldsymbol{K}$ is always non-singular, $\boldsymbol{u}$ can be eliminated from equations (5), and since also relations (4) hold, the following relations are obtained:

$$\boldsymbol{\varepsilon}_t = -(\boldsymbol{G}^T\overline{\boldsymbol{K}}^{-1}\boldsymbol{G})\boldsymbol{s}_t + (\boldsymbol{G}^T\overline{\boldsymbol{K}}^{-1}\overline{\boldsymbol{p}}_t + \boldsymbol{h}) = \boldsymbol{F}\,\boldsymbol{s}_t + \boldsymbol{\varepsilon}_{0t},$$

$$\boldsymbol{\varepsilon}_t \geq \boldsymbol{0}, \qquad -\boldsymbol{s}_t \geq \boldsymbol{0}, \qquad \boldsymbol{s}_t^T\boldsymbol{\varepsilon}_t = \boldsymbol{0} \tag{6}$$

where $\boldsymbol{F}$ is the $m \times m$ influence matrix of the reactions of the bonds to their corresponding fictitious strains, and $\boldsymbol{\varepsilon}_{ot}$ is the m-vector of the fictitious strains of the bonds due to the external loading. The matrix $\boldsymbol{F}$ is non-singular, thus $\boldsymbol{D} = \boldsymbol{F}^{-1}$ exists, and relations (6) can be written as:

$$\boldsymbol{s}_t = \boldsymbol{D}\boldsymbol{\varepsilon}_t + \boldsymbol{D}\boldsymbol{\varepsilon}_{o\,t} = \boldsymbol{D}\boldsymbol{\varepsilon}_t + \boldsymbol{s}_{o\,t}, \quad \boldsymbol{\varepsilon}_t \geq \boldsymbol{0}, \quad -\boldsymbol{s}_t \geq \boldsymbol{0}, \quad \boldsymbol{s}_t^T \boldsymbol{\varepsilon}_t = 0 \tag{7}$$

where $\boldsymbol{D}$ is the $m \times m$ influence matrix of fictitious strains of the bonds to their corresponding reactions, and $\boldsymbol{s}_{ot}$ is the m-vector of the reactions of the bonds due to the external loading.

The relations (6) (resp. the relations 7), which constitute a Linear Complementarity Problem (L.C.P.) give the solution of the problem.

Here for the numerical solution the relations (7) are used. The matrix $\boldsymbol{D}$ and the vector $\boldsymbol{s}_{ot}$ are not calculated through matrix invertion, but by using their physical meaning (Doudoumis and Mitsopoulou 1988) and the L.C.P. is solved by the Lemke's algorithm.

NUMERICAL EXAMPLES

As an example of unilateral behaviour of a structure under seismic loading, the five-storey building of figure 1 was studied. The strucrure is founded on stiff soil through continoous foundation beams. The structure is subjected to static vertical loads in conjuction with dynamic seismic excitation for which the accelerograms of Pacoima, Taft and El Centro earthquakes were used.

In figure 2 the bending moments at the base of the core during the Pacoima earthquake are shown for unilateral and bilateral structural behaviour. In figure 3 the maximum values of the bending moments at the base of the core and the columns 14 and 15 are shown, for the above mentioned earthquakes. The presented here sample of results shows that the structures with unilateral contact support conditions (if partial uplift occurs during an earthquake) take special dynamic characteristics and the response values could drastically change from the response values of the corresponding bilateral structures.

A systematic parametric analysis has been carried out (Zervas 1993), by which it has become clear that generally there is a decrease of the response values at the unilateral structure. But it should be noted that for some special cases a significant increase of the response values of the unilateral structure may also occur.

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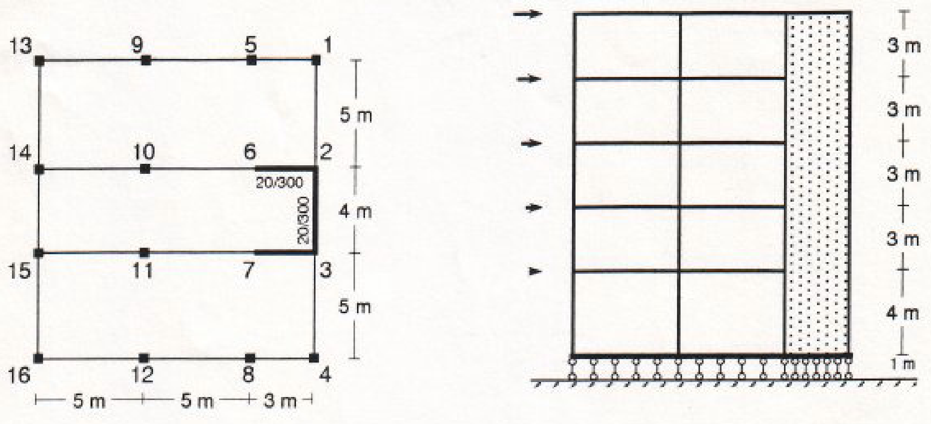

earthquake direction x-x

COLUMNS (section):			BEAMS: J=0.005 m4
Floor	Corner	Central	Foundation BEAM: J=0.100 m4
4-5-6	30x30	40x40	Vertical load per floor: 1.0 t/m2
2-3	35x35	45x45	
1	40x40	50x50	

Figure 1: Plan view and vertical section of a five-storey building

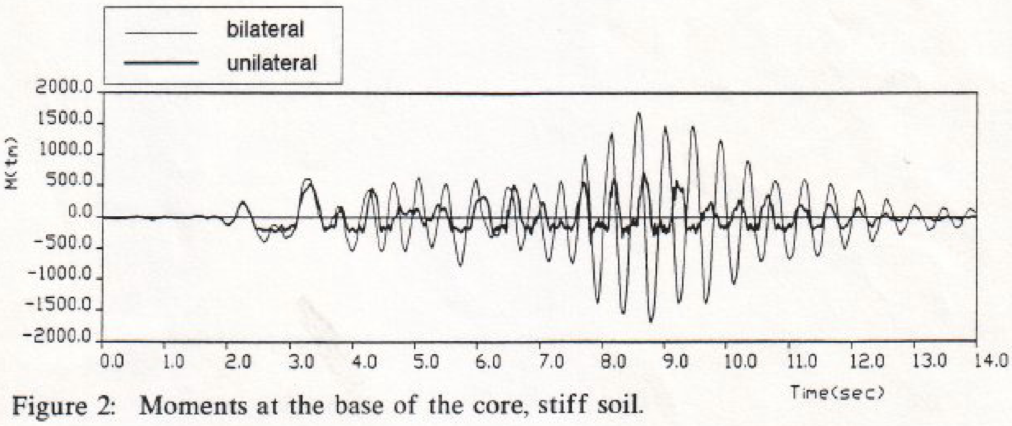

Figure 2: Moments at the base of the core, stiff soil.

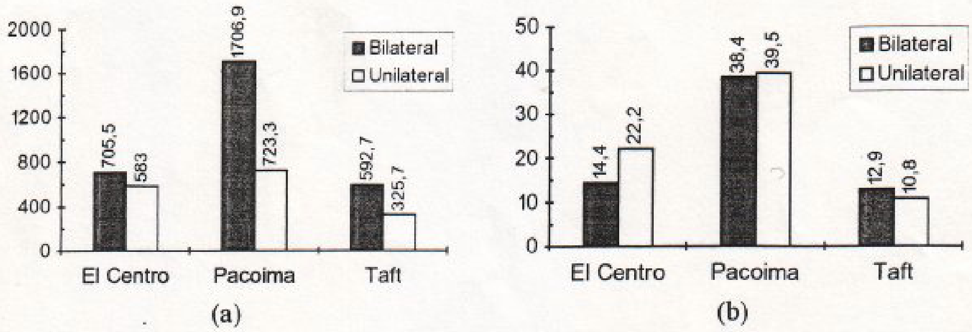

Figure 3: a) Maximum moments (tm) at the base of the core
b) Maximum moments (tm) at the base of the columns 14 and 15