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Seismic response of three-dimensional uplifting structures

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ABSTRACT : In the paper the seismic response of two-dimensional and three-dimensional structures with surface foundation is studied, for the case in which some parts of the foundation may uplift during the earthquake ("unilateral" contact conditions). The analyses are carried out by a computer program based on an algorithm proposed by the authors. Due to the variable support conditions the problem is nonlinear, but at each time step only the unknown displacements at the foundation are considered by using an elimination technique of the internal degrees of freedom. The contact-impact conditions at the foundation are also properly taken into account. A parametric numerical analysis of several 2D and 3D structures under characteristic excitations is carried out. The response of each structure is studied for unilateral and also bilateral (fixed) contact conditions at the foundation. The structures are subjected to static vertical loads in conjunction with the dynamic seismic excitation. Some interesting results and conclusions obtained from the parametric analysis of the seismic response of buildings that may uplift during an earthquake are presented.

1 FORMULATION OF A DISCRETE MATHEMATICAL MODEL

In order to formulate the discrete mathematical model of the problem we assume the following:

a. The displacements and the strains are infinitesimal.

b. The elastic structure is discretized by the finite element method. The $6n$-vector $\boldsymbol{u}$ of the nodal displacements (three deflections and three rotations per node) is the unknown vector of the problem

c. The contact between the structure and the soil is frictionless and is localized at the m nodes of the boundary Γ_s (for which the unilateral contact conditions hold). We consider first the discretized structure Ω_o for which only the bilateral (if any) kinematical conditions hold. The dynamic equations of equilibrium of this structure at any time are written as:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{p}(t) \qquad (1)$$

In (1) $\boldsymbol{K}$ is the $6n \times 6n$ stiffness matrix, $\boldsymbol{M}$ is the $6n \times 6n$ positive definite mass matrix, $\boldsymbol{C}$ is the $6n \times 6n$ damping matrix, $\boldsymbol{p}(t)$ is the $6n$ vector of the nodal forces, and $\dot{\boldsymbol{u}}$, $\ddot{\boldsymbol{u}}$ the $6n$ vectors of the nodal velocities and accelerations.

As long as contact-impact does not take place at any boundary point, the motion is fully described by the linear differential equations (1) and by the "initial" conditions at time $t=t_A$: $u=u_A$ and $\dot{u}=\dot{u}_A$. The "initial" conditions are introduced at time $t=0$ and at any time step t_A for which discontinuity of the velocities occurs due to contact-impact.

For the solution of the problem also an appropriate time discretization algorithm is used. Here the simple one-step algorithm of implicit type proposed by Zienkiewicz, Wood and Taylor (Zienkiewicz 1980) is used. On the basis of this algorithm the differential equations of motion (1) of the system Ω_o are converted to a set of algebraic equations which for the discrete time interval $(t-\Delta t)\div t$ takes the form:

$$\overline{\boldsymbol{K}} \cdot \boldsymbol{u}_t = \overline{\boldsymbol{p}}_t \tag{2}$$

After the above discretization, the unilateral contact conditions at the boundary Γ_s are introduced. At the m pairs of the discrete contact points, fictitious semi-rigid unilateral bonds with infinitesimal size are introduced, which can carry only compressive stresses. Each bond i has a direction normal to the contact surface and connects the adjacent nodes k and l. Denoting by:

s_i the stress of the bond i (normal contact reaction at the pair i),

ε_i the imposed strain of the bond i corresponding to s_i (relative displacements of the node pair k and l),

the following relations hold:

$$-s_i \geq 0, \quad \varepsilon_i = \boldsymbol{G}_i^T \boldsymbol{u} \geq 0, \quad -s_i \cdot \varepsilon_i = 0 \tag{3}$$

where

$\boldsymbol{G}_i^T = [\boldsymbol{o} \ldots \boldsymbol{g}_{ik} \ldots \boldsymbol{g}_{il} \ldots \boldsymbol{o}]$

is the $1 \times n$ strain-displacement matrix of the bond i connecting nodes k and l,

$-\boldsymbol{g}_{ik} = \boldsymbol{g}_{il} = \boldsymbol{g}_i = [n_{ix}, n_{iy}, n_{iz}, 0, 0, 0]$

and $n_i = [n_{ix}, n_{iy}, n_{iz}]$ is the unit vector of the direction cosines which is normal to the contact surface at the location of the bond i. For the total number of the m unilateral bonds, at the time step t, the following matrix relations can be written:

$$\boldsymbol{\varepsilon} = \boldsymbol{G}^T \boldsymbol{u} \geq \boldsymbol{0}, \quad -\boldsymbol{s} \geq \boldsymbol{0}, \quad \boldsymbol{s}^T \cdot \boldsymbol{\varepsilon} = \boldsymbol{0} \tag{4}$$

using the vectors: $s_t = [s_{1t} \ldots s_{mt}]$ $\varepsilon_t = [\varepsilon_{1t} \ldots \varepsilon_{mt}]$, and

$\boldsymbol{G}^T = [\boldsymbol{G}_1^T \quad \boldsymbol{G}_2^T \ldots \boldsymbol{G}_m^T]$. At

The unilateral contact kinematic conditions $\boldsymbol{\varepsilon}_t = \boldsymbol{G}^T \boldsymbol{u}_t$ will be taken into account together with the equations of dynamic equilibrium (2), throught the technique of Langrange multipliers. Accordingly to this technique if (2) are the dynamic equilibrium equations without any constrains, $\boldsymbol{G}^T \boldsymbol{u}_t - \boldsymbol{\varepsilon}_t = \boldsymbol{0}$ are the kinematic constraints, and $\boldsymbol{s}_t$ are the reactions corresponding to the kinematic constraints, then (see e.g. Washizu 1975):

$$\overline{\boldsymbol{K}} \boldsymbol{u}_t + \boldsymbol{G} \boldsymbol{s}_t = \overline{\boldsymbol{p}}_t, \quad \boldsymbol{G}^T \boldsymbol{u}_t = \boldsymbol{\varepsilon}_t \tag{5}$$

Since the matrix $\boldsymbol{K}$ is always non-singular, $\boldsymbol{u}$ can be eliminated from equations (5), and since also relations (4) hold, the following relations are obtained:

$$\begin{gathered} \boldsymbol{\varepsilon}_t = -(\boldsymbol{G}^T \overline{\boldsymbol{K}}^{-1} \boldsymbol{G}) s_t + (\boldsymbol{G}^T \overline{\boldsymbol{K}}^{-1} \overline{\boldsymbol{p}}_t) = \boldsymbol{F} s_t + \varepsilon_{0t}, \\ \boldsymbol{\varepsilon}_t \geq \boldsymbol{0}, \quad -s_t \geq \boldsymbol{0}, \quad s_t^T \boldsymbol{\varepsilon}_t = 0 \end{gathered} \tag{6}$$

where $\boldsymbol{F}$ is the $m \times m$ influence matrix of the reactions of the bonds to their corresponding fictitious strains, and $\boldsymbol{\varepsilon}_{ot}$ is the m-vector of the fictitious strains of the bonds due to the external loading. The matrix $\boldsymbol{F}$ is non-singular, thus $\boldsymbol{D} = \boldsymbol{F}^{-1}$ exists, and relations (6) can be written also as:

$$\begin{gathered} s_t = \boldsymbol{D} \varepsilon_t + \boldsymbol{D} \varepsilon_{o\,t} = \boldsymbol{D} \varepsilon_t + s_{o\,t}, \\ \boldsymbol{\varepsilon}_t \geq \boldsymbol{0}, \quad -s_t \geq \boldsymbol{0}, \quad s_t^T \boldsymbol{\varepsilon}_t = 0 \end{gathered} \tag{7}$$

where $\boldsymbol{D}$ is the $m \times m$ influence matrix of fictitious strains of the bonds to their corresponding reactions, and $\boldsymbol{s}_{ot}$ is the m-vector of the reactions of the bonds due to the external loading.

The relations (6) (resp. the relations 7), which constitute a Linear Complementarity Problem (L.C.P.) give the solution of the problem. Here for the numerical solution the relations (7) are used. The matrix $\boldsymbol{D}$ and the vector $\boldsymbol{s}_{ot}$ are not calculated through matrix invertion, but by using their physical meaning (Doudoumis and Mitsopoulou 1988). The problem is solved by the Lemke's algorithm (Murty 1988) and thus the final condition of each nodal pair of points on the contact surface is obtained.

At this time if a point k of the foundation comes into contact with the adjucent point l of the soil, the velocity of the point k changes due to impact and proper 'initial' conditions must be imposed at points k-l (Mitsopoulou 1983). The velocity u_i^+ of the point k after impact may become zero ($u_i^+ = e \cdot u_i^- = 0$) in case of perfectly plastic impact, it may be reversed ($u_i^+ = e \cdot u_i^- = -u_i^-$) in case of perfectly elastic impact, or it may take any intermediate value (restitution factor $e = 0 \div 1$). In the paper perfectly plastic impact is considered. We must say here, that the numerical solution of many problems has proved that the nature of impact (the value of the restitution factor) does not affect significantly the results.

2 PARAMETRIC ANALYSIS

A computer program based on the proposed method has been developed by the authors and a systematic parametric analysis of the seismic response of buildings that may uplift during a dynamic excitation has been carried (Zervas 1993). Here we present some characteristic results and conclusions obtained from the parametric analyses.

2D and 3D structures are studied, (see for example the structures shown in fig. 1). The structures are founded on elastic soil (Winkler assumption) through continuous foundation beams. The response of each structure is studied for unilateral and also for bilateral (fixed) contact conditions at the base of the footing. The structures are subjected to a dynamic loading in conjuction with the static vertical loads. For a great number of structures numerical analyses were performed, first using harmonic sinusoidal excitations for which the frequency and the amplitude of the excitation were the variable parameters, and then using different earthquake accelerograms (El Centro, Taft, Pacoima Dam, of Thessaloniki). For the structures of figure 1 some characteristic results and comparisons of their response are presented in the following. The response values are taken for unilateral contact conditions at the foundation (the structure is allowed to uplift), and for bilateral contact conditions at the foundation (the structure has fixed support conditions at the base).

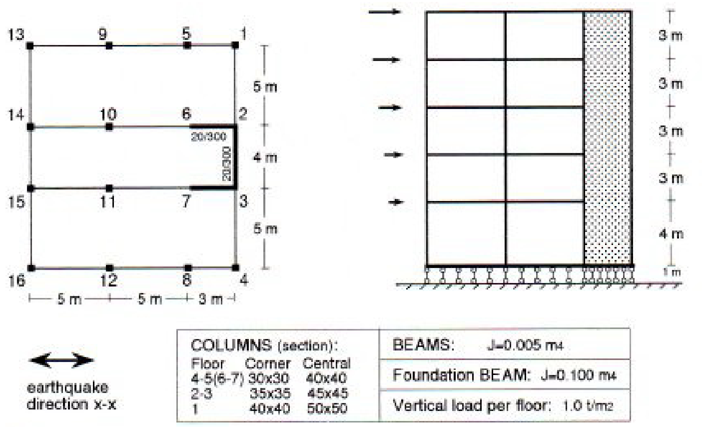


Figure 1 Plan view and vertical section of the five-storey (seven-storey) structure.

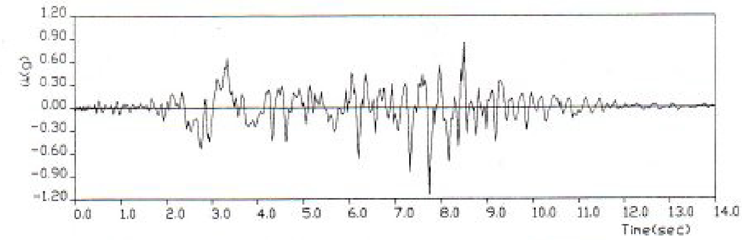
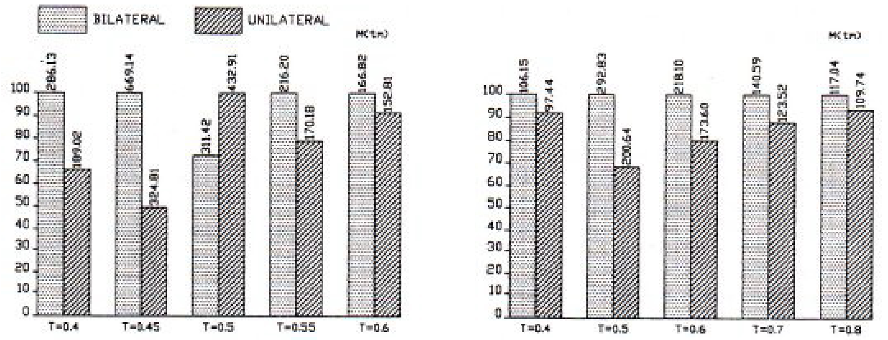


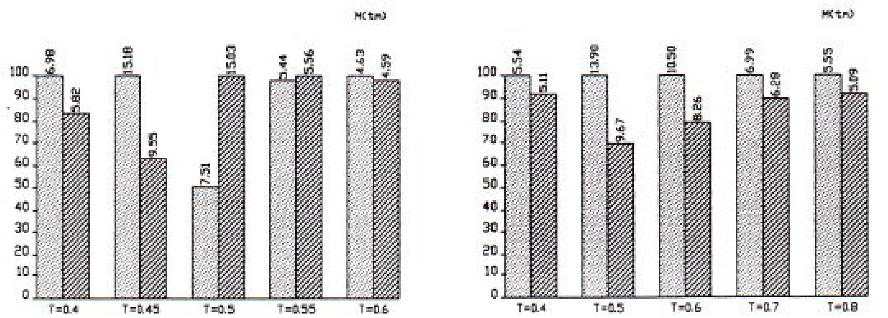
Figure 2 The ground accelerations of the Pacoima Dam S16E accelerogram.



Stiff soil

Soft soil

Figure 3 The maximum dending moments at the base of the core of the five-storey structure for different values of the period of the sinusoidal excitation, for stiff and soft soil.



Stiff soil

Soft soil

Figure 4 The maximum dending moments at the base of the columns 14,15 of the five-storey structure for different values of the period of the sinusoidal excitation, for stiff and soft soil.

The influence of the frequency of the excitation on the bending moments at characteristic parts of the five-storey structure for stiff and soft soil conditions is shown in figures 3 and 4. Generally there is a decrease at the responce values (displacements, bending moments etc.) of the uplifting structure in relation with the coresponding values of the fixed support structure. When the period T of the excitation equals the first natural period T1 of the structure (T1=0.45 sec for stiff soil, T1=0.5 sec for soft soil) the maximum displacements and stresses for bilateral contact conditions are obtained. The maximum bending moment at the base of the core is 669 tm ($M^b_{0.45}$=669tm), and at the base of columns 14,15 is 15.2tm ($M^b_{0.45}$=15.2tm), while for unilateral contact conditions the corresponding values are $M^u_{0.45}$=324tm and $M^u_{0.45}$=9.5tm. In this case the lowest relative values of the bending moments etc. of the uplifting structure compared with the values of the fixed support structure are taken (for example $M^u_{0.45}$=0.484 $M^b_{0.45}$). The maximum displacements and stresses of the uplifting structure are taken for a period T of the excitation greater than the first natural period T1 of the considered structure. We could say that this is the 'natural period' of the uplifting

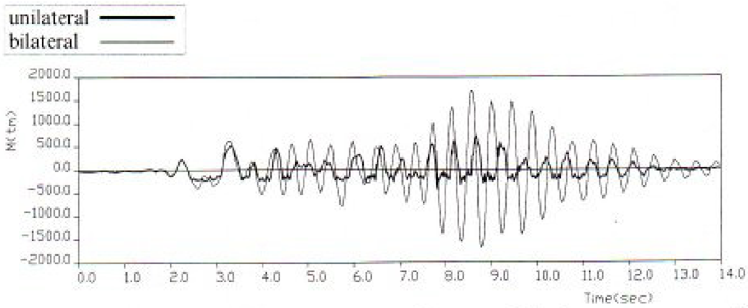


Figure 5 The bending moments at the base of the core of the five-storey structure for the Pacoima earthquake accelerogram for stiff soil conditions.

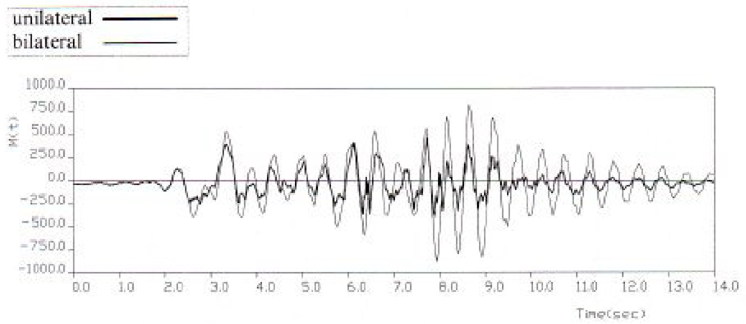


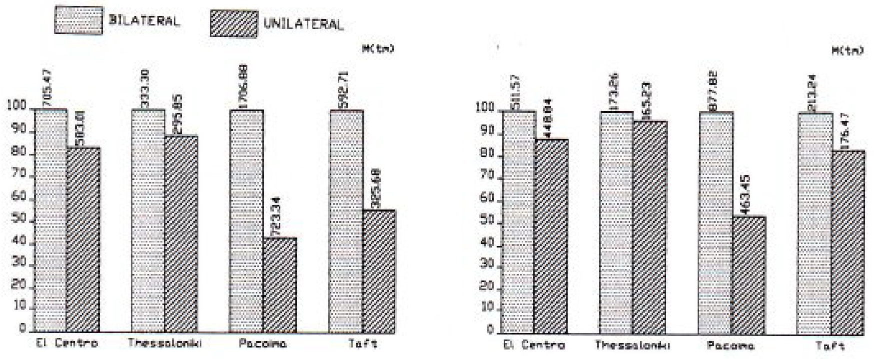
Figure 6 The bending moments at the base of the core of the five-storey structure for the Pacoima earthquake accelerogram for soft soil conditions.

structure while it is known that such a quantity does not exist. For example for stiff soil and T=0.50 sec, the maximum bending moment at the base of the core of the uplifting structure is 432tm ($M^u_{0.50}$=432tm) a value which is greater than the corresponding value $M^b_{0.50}$= 311tm taken for bilateral contact conditions. In this case the upper relative values of the bending moments etc. of the uplifting structure compared with the values of the fixed support structure are taken (for example $M^u_{0.50}=1.39\ M^b_{0.50}$). But it is also observed that the maximum bending moments of the structure with fixed boundary conditions are greater than the maximum bending moments of the uplifting structure (669>432, 15.2>15, for stiff soil, 292.8>200.6, 13.9>9.7 for soft soil).

From the parametric analysis it was proved that the above observations are true for all kinds of structures and for any values of excitation amplitudes and soil conditions, which means that when a structure is allowed to uplift it has better seismic response.

In figures 5 and 6 the change of the bending moments at the base of the core during an earthquake are shown. The Pacoima Dam earthquake accelerogram is used. Except for the first four seconds of the excitation for which small ground acceleration values are given, there is a considerable difference between the bending moments of the uplifting structure and the fixed support structure.

In figure 7 the difference between the maximum values of the bending moments of the fixed support structure and the bending moments of the uplifting structure for the different earthquake accelerograms used, is clear cut.



Stiff soil   Soft soil

Figure 7 The maximum dending moments at the base of the core of the five-storey structure for the El Centro, Thessaloniki, Pacoima, Taft earthquake accelerograms, for stiff and soft soil.

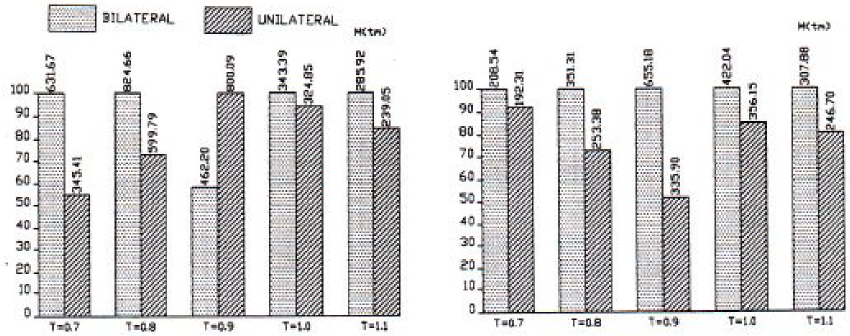


Figure 8 The maximum dending moments at the base of the core of the seven-storey structure for different values of the period of the sinusoidal excitation, for stiff and soft soil.

Examining figure 8 we arrive, for the seven-storey building of figure 1, at similar conclusions to those obtained from figures 3 and 4, for the five-storey building.

In figure 9 the change of the bending moments at the base of the core for the Pacoima Dam earthquake accelerogram is shown. Only for the first 2.5 seconds of the excitation the responce for unilateral and bilateral support conditions is similar, after this time there is a considerable difference between the bending moments. Contrary to what happens with the five-storey building, here the bending moments are greater for the uplifting structure.

Finally in figure 10 the difference between the maximum values of the bending moments of the fixed structure and the bending moments of the uplifting structure for the different earthquake accelerograms is shown. Here we have the only case (for stiff soil and Pacoima Dam accele-rogram) in which the uplifting structure behaves worse than the fixed structure.

The parametric analysis has shown that the structures with unilateral contact support condi-tions (if partial uplift occurs during the earth-quake) take special dynamic characteristics and the response values change significantly. In the paper is shown that the different parameters, like

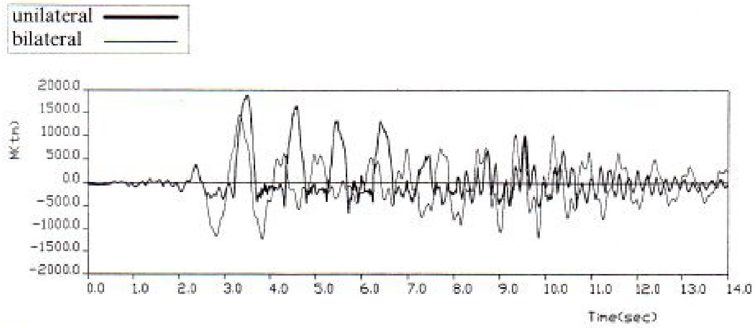


Figure 9 The bending moments at the base of the core of the seven-storey structure for the Pacoima earthquake accelerogram for "stiff" soil conditions.

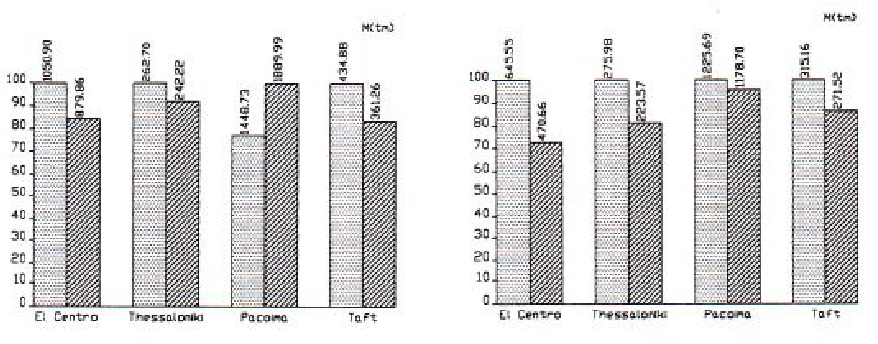


Figure 10 The maximum dending moments at the base of the core of the seven-storey structure for the El Centro, Thessaloniki, Pacoima, Taft earthquake accelerograms, for stiff and soft soil.

the frequency range of the excitation, the intensity of the excitation and the stiffness of the soil, affect drastically the results. Generally we can say that, especially for soils which are not very stiff, the response values of the uplifting structures are lower than the response values of the same structures considered with fixed (no uplifting) support conditions, because in the uplifting case the resonance phenomena are unusual to occur.

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